Università di Bologna – Campus di Rimini – Corso di laurea in Farmacia e CQPS

Esame MATEMATICA 14/04/2014 – Docente: Stefano Bordoni

STUDENTE: ________________________; CORSO di LAUREA: ___________

MATRICOLA: _______; N° documento: ____________; FIRMA: ___________

1. Calcolare $\log_9(3)$, $\log_x(\frac{1}{x^2})$, $\log_x(\sqrt[2]{x^3})$, $\frac{(n+1)!}{(n-1)!\cdot n^2}$ [4]

2. Risolvere: $3^x \geq 9$, $\log_2(1-x^2) < 0$, $\log_{\frac{1}{3}}(3-x) < -2$ [4]

3. Calcolare la probabilità che una coppia di criceti generi 3 maschi e 5 femmine in una nidiata di 8 figli. [3]

4. Eseguire lo studio globale della funzione $y = f(x) = \sqrt[3]{x+1}$, cioè determinare dominio, grafico e codominio. [4]

5. Determinare la funzione derivata e una funzione primitiva della stessa funzione $y = f(x) = \sqrt[3]{x+1}$. [3]

x x x x x x x

6. Eseguire lo studio analitico della funzione $y = \ln(x^2 - 1)$ (è accettabile anche uno studio globale accuratamente giustificato). [8]

7. Data la funzione $y = f(x) = e^x - 1$ e la sua funzione inversa $y = g(x)$, determinare dominio, grafico e codominio di entrambe.

Controllare se $g(x)$ soddisfa le ipotesi del teorema di Weierstrass sull'intervallo $[0\ ;\ e-1]$.

In caso positivo, determinare il massimo e il minimo della funzione su tale intervallo. [4]

8. Risolvere graficamente la disequazione $x^2 < \sqrt{|x|}$. [2]

# MATEMATICA – FARMACIA e CQPS

## 14/04/2014

1

- $9^{\square} = 3 \qquad (3^2)^{\square} = 3^1 \qquad 2 \cdot \square = 1 \qquad \square = \frac{1}{2}$

- $x^{\square} = \frac{1}{x^2} \qquad x^{\square} = x^{-2} \qquad \square = -2$

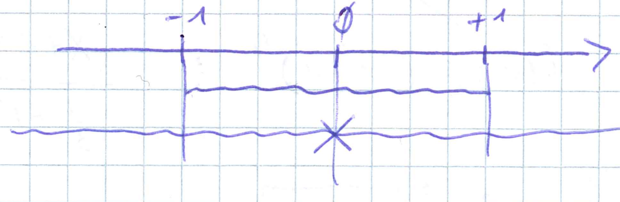
- $x^{\square} = \sqrt[2]{x^3} \qquad x^{\square} = x^{3/2} \qquad \square = 3/2$

- $\frac{(n+1)\cdot \cancel{n}\,\cancel{(n-1)}!}{n \cdot \cancel{n} \cdot \cancel{(n-1)}!} = \frac{n+1}{n}$

2

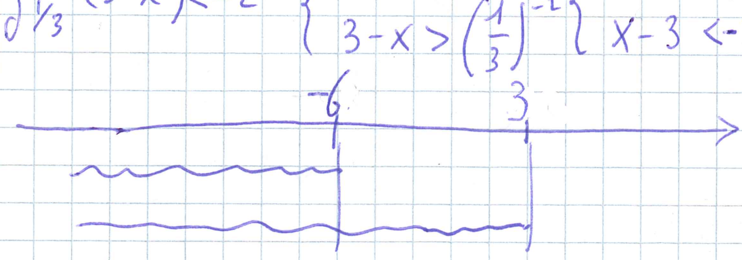
- $3^x \geq 9 \qquad 3^x \geq 3^2 \qquad x \geq 2$

- $\log_2 (1-x^2) < \phi \qquad \begin{cases} 1-x^2 > \phi \\ 1-x^2 < 2^{\phi} \end{cases} \quad \begin{cases} x^2-1 < \phi \\ x^2-1 > -1 \end{cases} \quad \begin{cases} x \in \left]-1; +1\right[ \\ x^2 > \phi \to \forall x \neq \phi \end{cases}$




$x \in \left]-1; \phi\right[ \cup \left]\phi; +1\right[$

- $\log_{1/3} (3-x) < -2 \qquad \begin{cases} 3-x > \phi \\ 3-x > \left(\frac{1}{3}\right)^{-2} \end{cases} \quad \begin{cases} x < 3 \\ x-3 < -9 \end{cases} \quad \begin{cases} x < 3 \\ x < -6 \end{cases}$

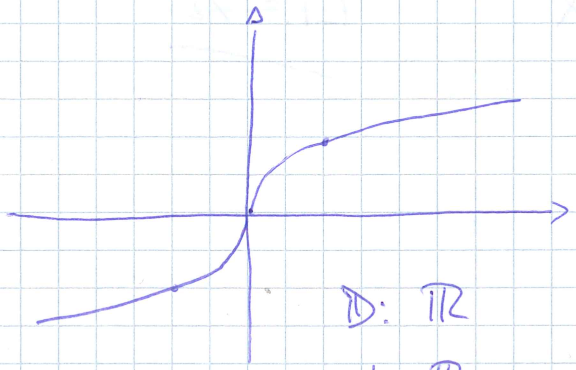



$x \in \left]-\infty; -6\right[$

3. $P(3M-5F) = \frac{\binom{8}{3}}{2^8} = \frac{\binom{8}{5}}{2^8} = \frac{1}{256} \cdot \frac{8!}{5!(8-5)!}$

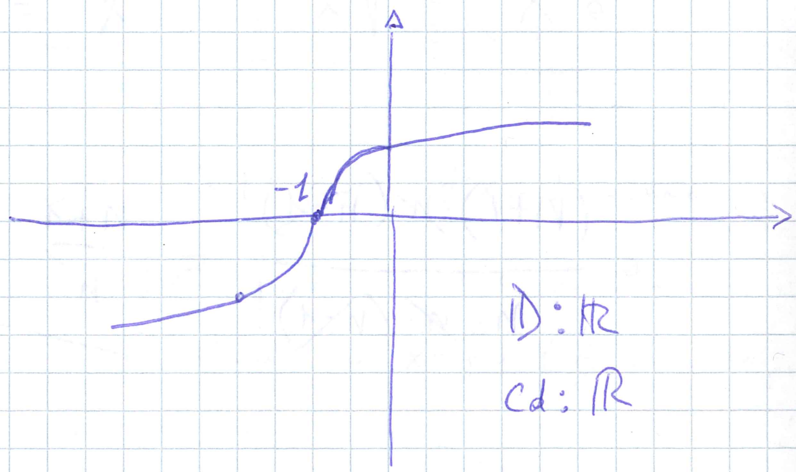
$= \frac{1}{\cancel{256}\ 32} \cdot \frac{\cancel{8}^{1} \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{5!}\ \cancel{3!}} = \frac{7}{32}$

4. $y = \sqrt[3]{x} \xrightarrow[1]{T_x} y = \sqrt[3]{x+1}$



$\mathbb{D}: \mathbb{R}$

$Cd: \mathbb{R}$




$\mathbb{D}: \mathbb{R}$

$Cd: \mathbb{R}$

5. • $y = f'(x) = \frac{1}{3}(x+1)^{1/3 - 1} \Longleftarrow y = f(x) = (x+1)^{1/3}$

$= \frac{1}{3}(x+1)^{-2/3} = \frac{1}{3\sqrt[3]{(x+1)^2}}$

• Primitive di $f(x) = (x+1)^{1/3}$ : $\frac{(x+1)^{\frac{1}{3}+1}}{\frac{1}{3}+1} + cost$

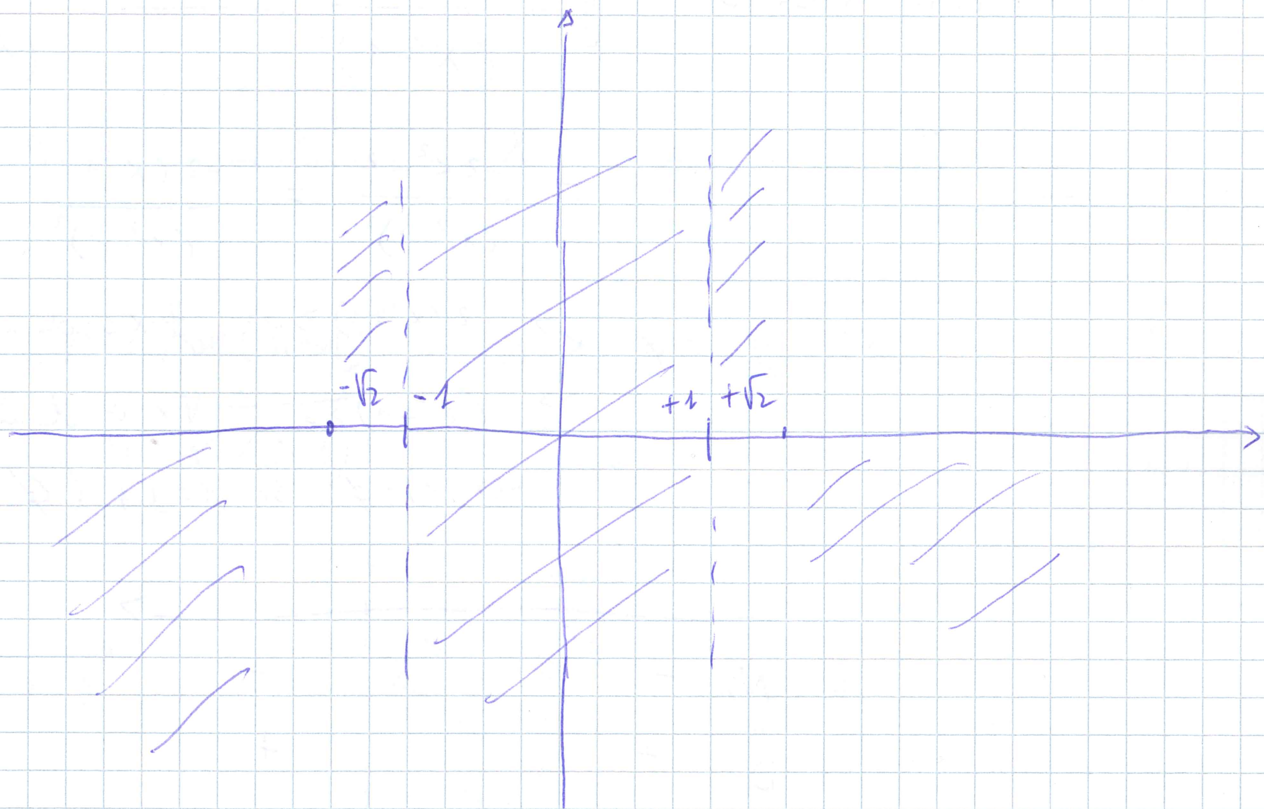
$\rightarrow \frac{(x+1)^{4/3}}{4/3} = \frac{3}{4}\sqrt[3]{(x+1)^4}$

6. Dominio: $x^2-1>0$ $\quad x\in\,]-\infty;-1[\,\cup\,]+1;+\infty[$

Invarianze x simmetrie: $f(x)=f(-x) \to$ PARI

Segno: $\ln(x^2-1)\geq 0$ $\quad x^2-1\geq e^0$ $\quad x^2-1\geq 1$

$x^2\geq 2$ $\quad x\in\,]-\infty;-\sqrt{2}[\,\cup\,]+\sqrt{2};+\infty[$




Limiti: $\lim\limits_{x\to-\infty}\ln(x^2-1)=\ln(+\infty)=+\infty$

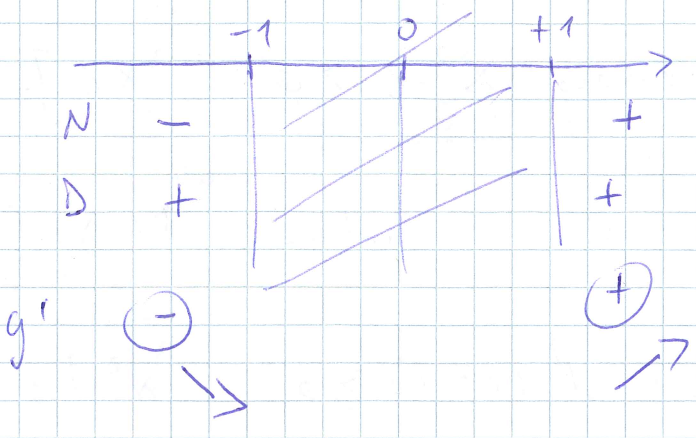
$\lim\limits_{x\to-1^-}\ln(x^2-1)=\ln[1^+-1]=\ln(0^+)=-\infty$

$\to$ x simmetrie $\lim\limits_{x\to+1^+}f(x)=-\infty$, $\lim\limits_{x\to+\infty}f(x)=+\infty$

Derivata: $y'(x)=\dfrac{2x}{x^2-1}$ $\quad x\in\,]-\infty;-1[\,\cup\,]+1;+\infty[$

? x: $y'\geq 0$ $\to$ $2x\geq 0$ $\quad x\geq 0$

$\to$ $x^2-1\geq 0$ $\quad x\in\,]-\infty;-1[\,\cup\,]+1;+\infty[$



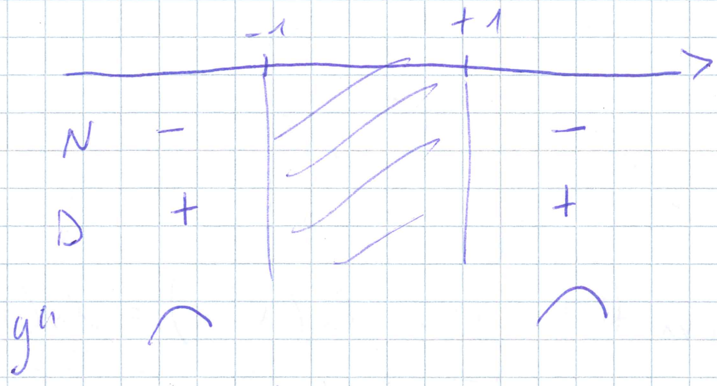

Derivata seconda: $y'' = \dfrac{2\cdot(x^2-1)-2x\cdot 2x}{(x^2-1)^2} = \dfrac{+2x^2-2-4x^2}{(x^2-1)^2}$

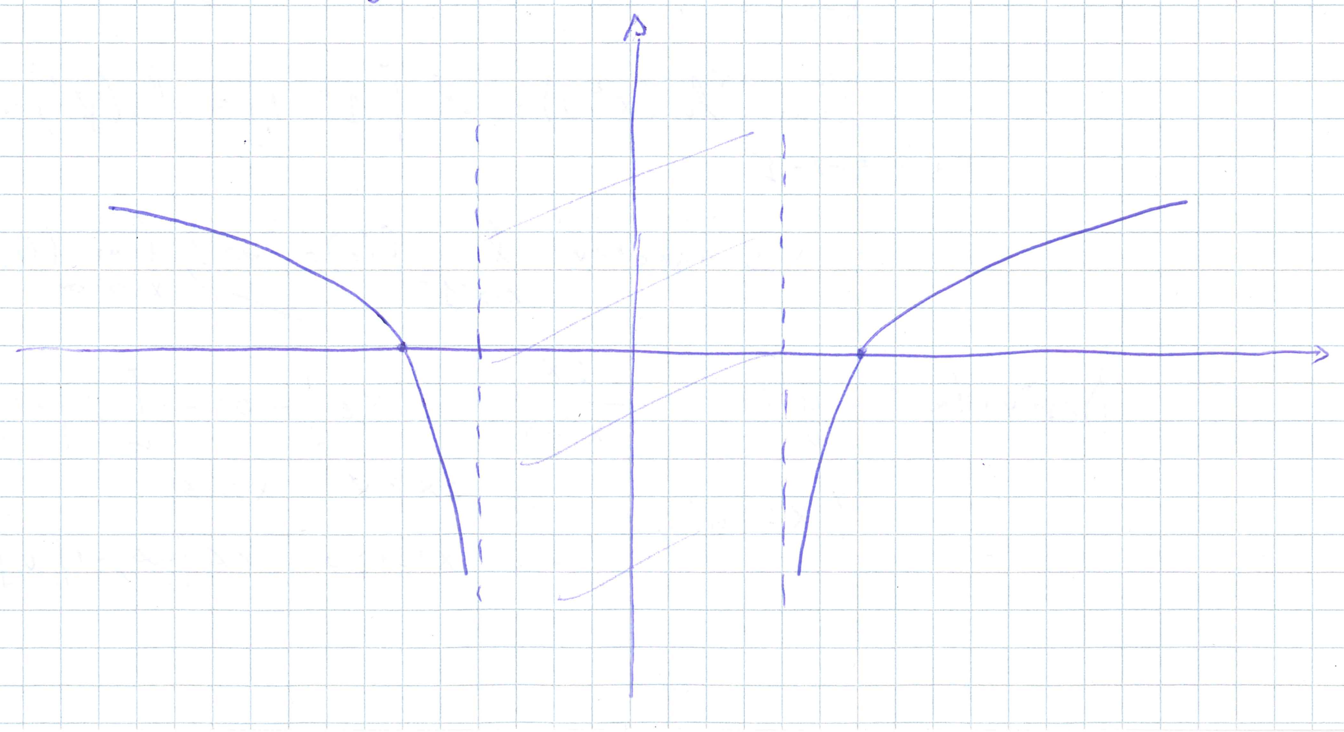
$= \dfrac{-2x^2-2}{(x^2-1)^2} = \dfrac{-2(x^2+1)}{(x^2-1)^2}$

? $x$: $y'' \geq \emptyset$ $\rightarrow$ $-2(x^2+1) \geq 0$ $\nexists x \in$ dominio

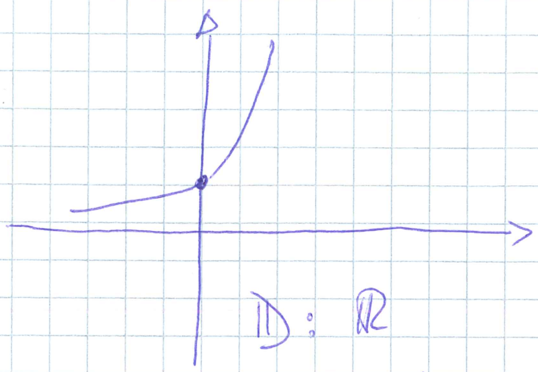
$\rightarrow$ $(x^2-1)^2 \geq 0$ $\forall x \in$ dominio

$(x^2-1) = \emptyset$ se $x = \pm 1$ ($\notin$ dominio)



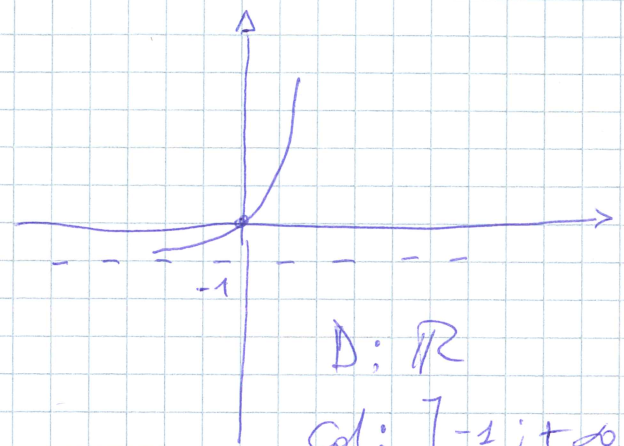



7. $y = e^x \xrightarrow{T_y} y = e^x - 1 \quad (y = f(x))$



D: $\mathbb{R}$

Cd: $\mathbb{R}^+$




D: $\mathbb{R}$

Cd: $]-1; +\infty[$

FUNZIONE INIETTIVA

$y = g(x) = f^{-1}(x) \longrightarrow x = e^y - 1 \qquad x+1 = e^y$

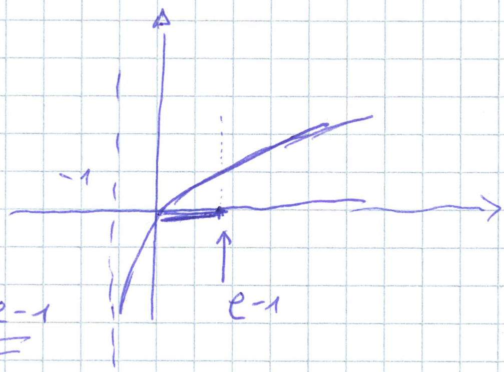
$\ln(x+1) = \ln(e^y) \qquad y = \ln(x+1)$

D: $]-1; +\infty[$

Cd: $\mathbb{R}$

La funzione $y = g(x)$ esiste ed è continua sull'intervallo $[\phi; e-1]$

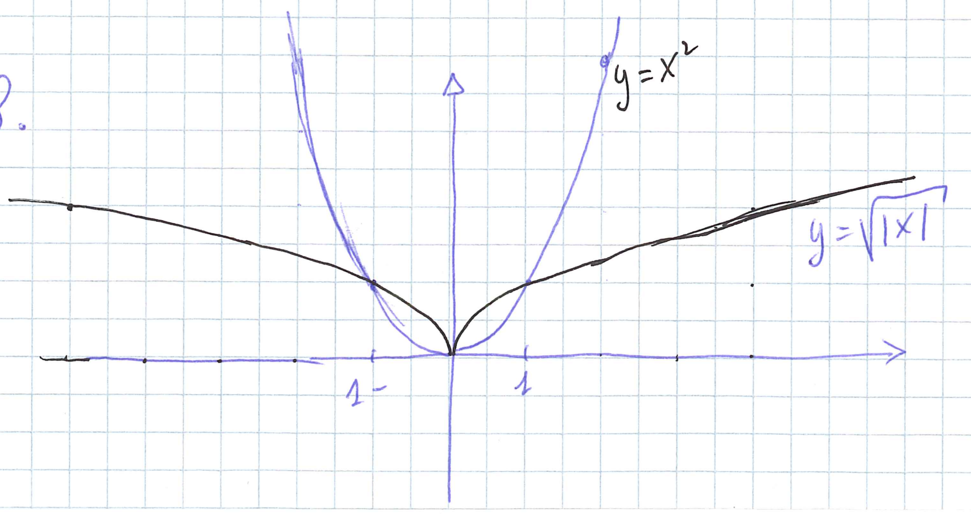
Essa quindi ammette MAX. e MIN.




MAX $= \ln(e-1+1) = \ln(e) = 1$ in $x = e-1$

MIN $= \ln(\phi+1) = \ln(1) = \phi$ in $x = \phi$

8.




$x \in ]-1; \phi[ \cup ]\phi; +1[$